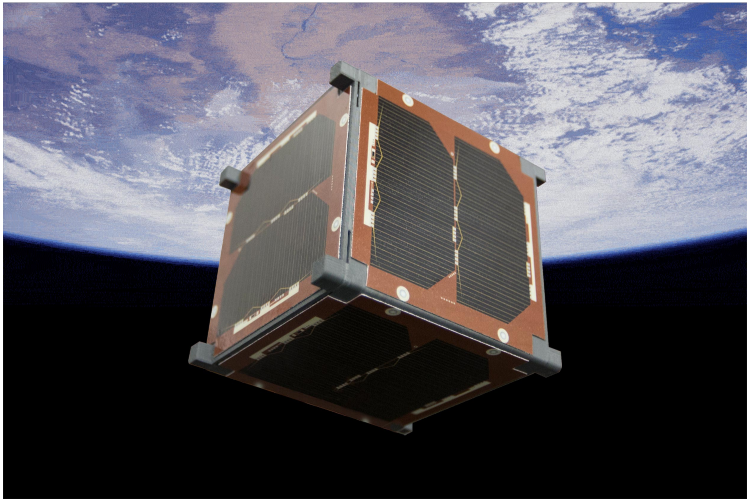
Synthesis of Digital Antenna Array Beam Forming

David Pánek

6. října 2023

Motivation - antenna for ground station



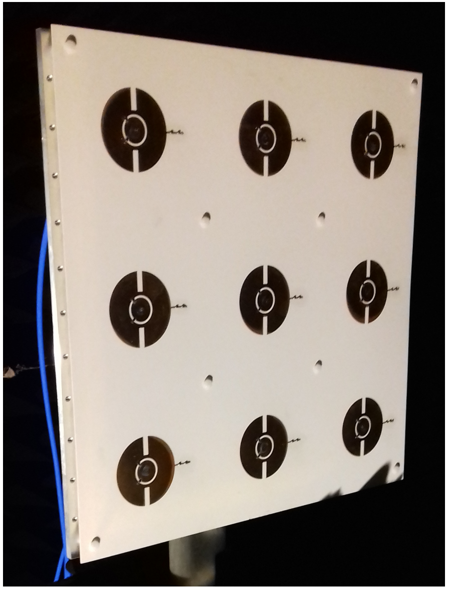
CubeSats

- Initiative to offer non-profit organizations to launch small satellites.
- Low-Earth-Orbit(LEO), attitude 250-900 km.
- Size must be within $0.1 \times 0.1 \times 0.1\,\mathrm{m}$
- Communication frequencies Ka-band(26-40 GHz), X-band(8-12 GHz), S-band (2-4 GHz)

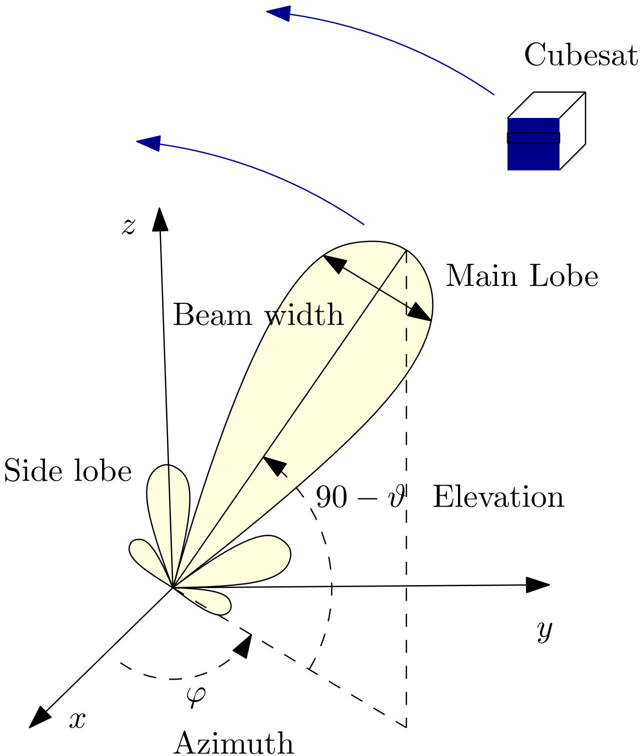
TEAM

- University of West Bohemia
  - Ing. Ivo Veřtát Ph.D. (VZLUSAT, hardware)
  - Ing. Michal Pokorný Ph.D. (hardware, measurement)
- Czech Technical University
  - prof. Ing. Miloš Mazánek, CSc. (project leader, antenna design, experiments)
  - doc. Ing. Jiří Masopust, CSc. (hardware, concepts, experiments design)
  - doc. Ing. Pavel Hazdra, Ph.D. (antenna design, CST simulations)
  - Ing. Jan Kraček (control synthesis, CST simulations)
  - Ing. Milan Švanda Ph.D. (experiment design)

Tracking parabolic antennas have some issues



Radiation pattern



Beam forming

Amplitudes and phases of the excitation currents that provide the desired radiation pattern.

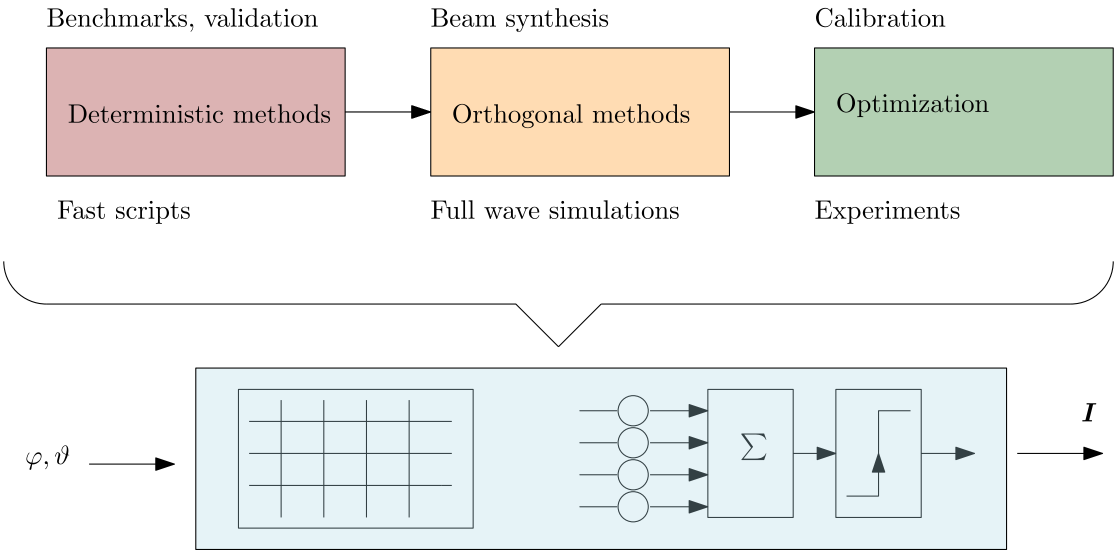
- Electric field in spherical coordinates

$$\boldsymbol{f}(\vartheta,\varphi) = \boldsymbol{f}_{\mathrm{e}}(\vartheta,\varphi) \cdot S(\vartheta,\varphi) \cdot M(\vartheta,\varphi), \quad (1)$$

- Array factor

$$S(\varphi,\vartheta) = \sum_{n=1}^{N}\sum_{m=1}^{M} \underline{I}_{nm} \exp\left[\mathrm{j} \cdot k_0 \cdot n \cdot d_x \sin(\theta)\cos(\varphi) + k_0 \cdot m \cdot d_y \sin(\vartheta)\sin(\varphi)\right], \quad (2)$$

Radiation patter synthesis



Deterministic methods

- Array factor is image of $\boldsymbol{I}$ in 2-D Discrete Fourier Transform

$$\boldsymbol{S}(\theta, \phi) = \sum_m \sum_n I[m, n] e^{-jk(md \sin(\theta) \cos(\phi) + nd \sin(\theta) \sin(\phi))}, \tag{3}$$

- can be seen also as a 2-D Z transform

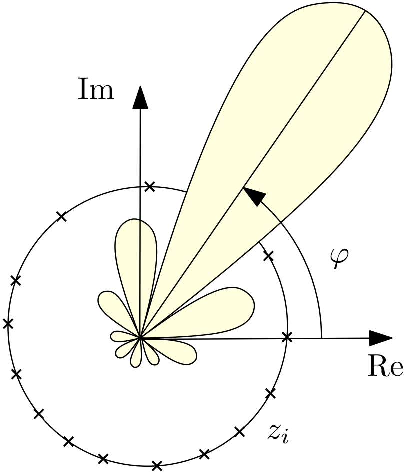
$$\boldsymbol{S}(z_1, z_2) = \sum_m \sum_n I[m, n] z_1^{-m} z_2^{-n}. \tag{4}$$

where

$$e^{-jkd \sin(\theta) \cos(\phi)} \leftrightarrow z_1^{-1} \tag{5}$$
$$e^{-jkd \sin(\theta) \sin(\phi)} \leftrightarrow z_2^{-1} \tag{6}$$

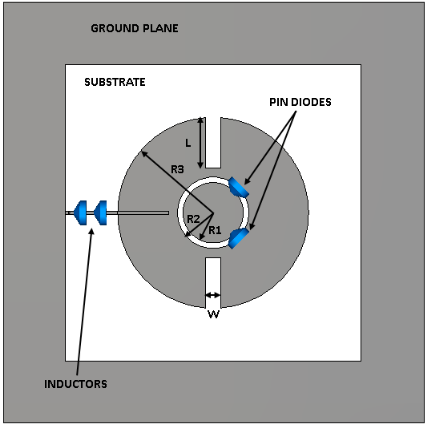
Deterministic methods

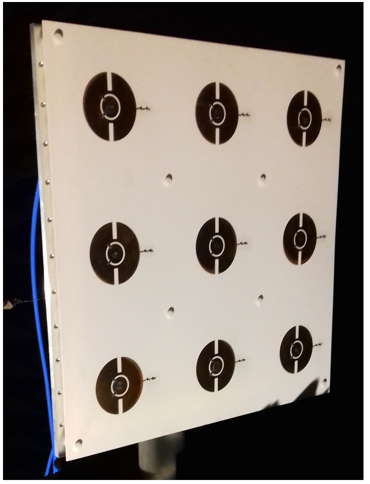
- Deterministic methods - are similar to FIR filter design
- Inverse Fourier transform
- Windowing
- Zero placement
- Dolph-Chyebyshev method



WHY IT DOESN'T WORK?

- There are physical reasons - low number of elements, mutual couplings





Orthogonal methods

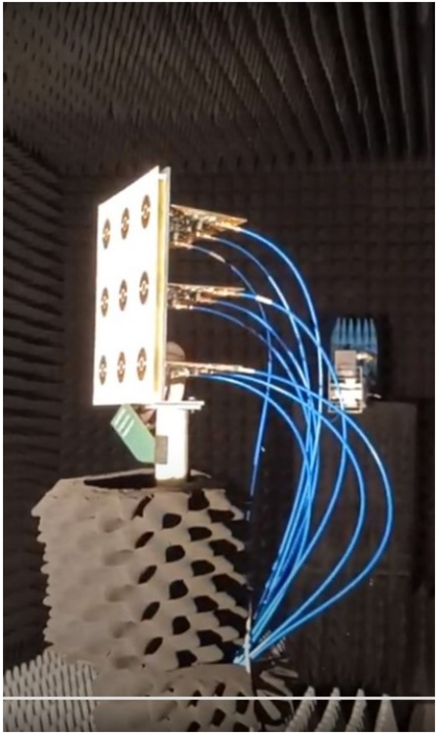
- Radiation pattern can be written in the form

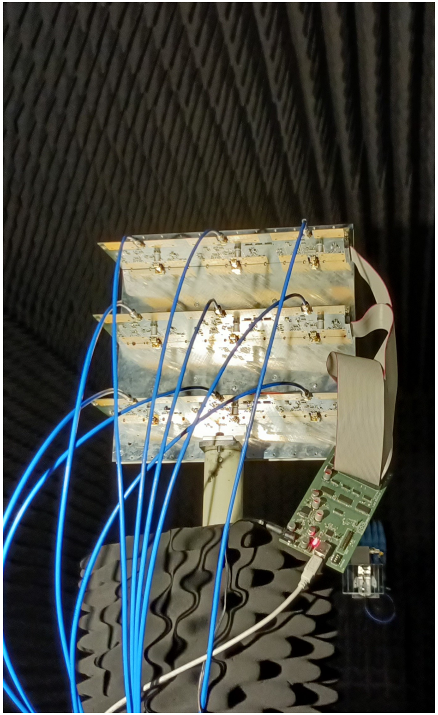
$$\boldsymbol{f}_g = \boldsymbol{f}_\psi \cdot \boldsymbol{I}, \tag{7}$$

- where $\boldsymbol{f}_g$ is the desired radiation pattern, $\boldsymbol{I}$ is a vector of excitation and $\boldsymbol{f}_\psi$ are basis functions,
- Optimal excitation can be expressed as

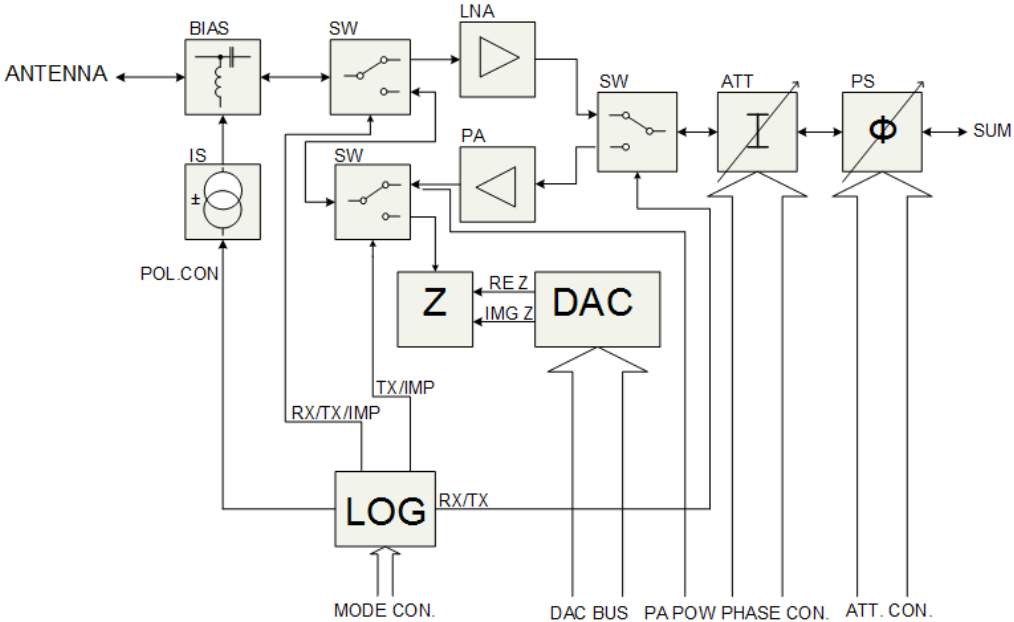
$$\hat{\boldsymbol{I}} = (f_\psi^T \cdot f_\psi)^{-1} \boldsymbol{f}_\psi^T \cdot \boldsymbol{f}_g, \tag{8}$$

Measurement





Why it doesn't work?



Optimization

- What we want? Maximal magnitude of the main lobe. Maximally narrow main lobe. Maximal suppression od side lobes.

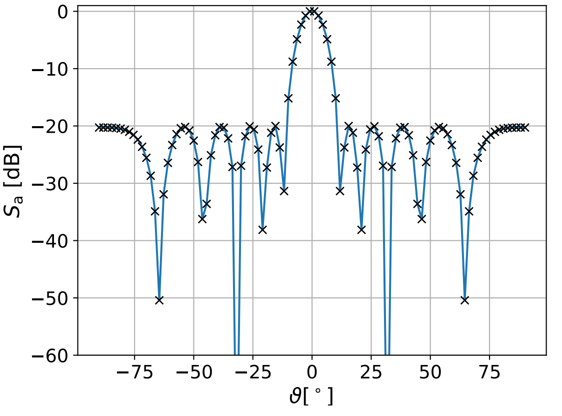
$$\hat{\boldsymbol{I}} = \arg\min \mathcal{F}(\boldsymbol{f}_g(\boldsymbol{I})) \tag{9}$$

- Goal functions

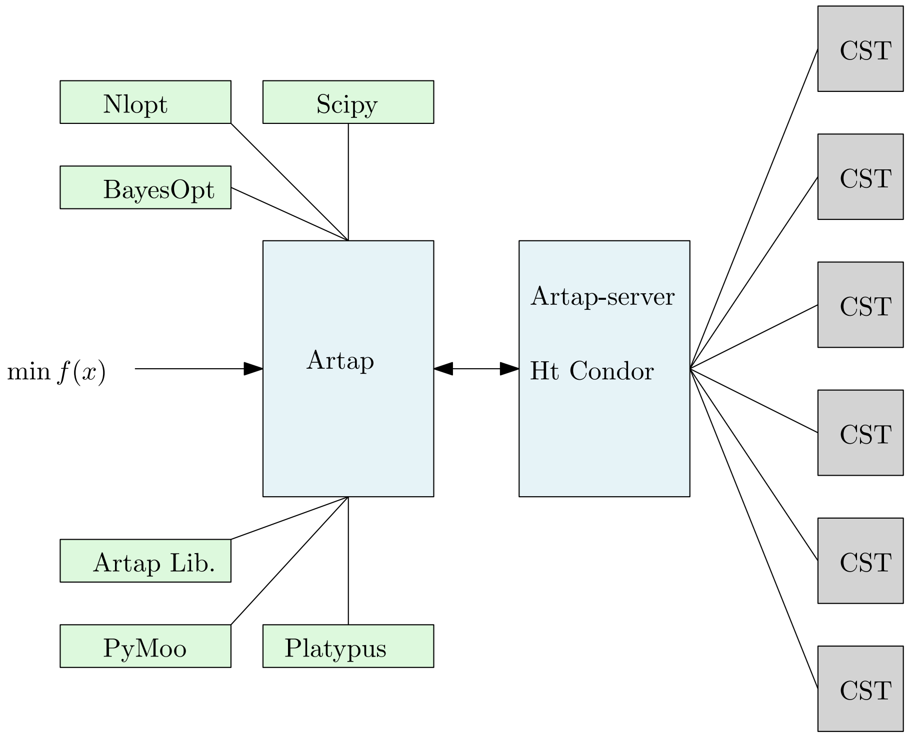
$$\mathcal{F}_1 = -|S(\vartheta_\mathrm{r}, \varphi_\mathrm{r})|, \tag{10}$$

$$\mathcal{F}_1 = |\hat{S}_1| - |\hat{S}_2|, \tag{11}$$

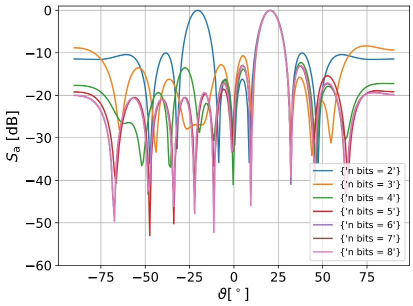
$$\mathcal{F}_1 = \|\hat{S} - \hat{S}_\mathrm{r}\|^2, \tag{12}$$



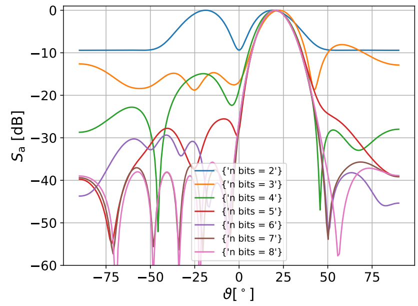
Artap


Nlopt
Scipy
BayesOpt
$\min f(x)$
Artap
Artap-server
Ht Condor
Artap Lib.
PyMoo
Platypus
CST
CST
CST
CST
CST
CST

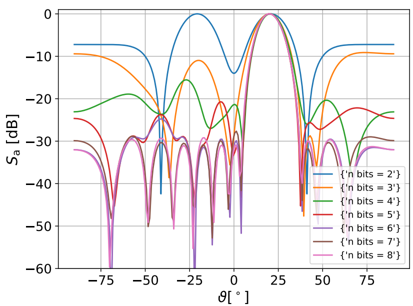
And what about quantization?



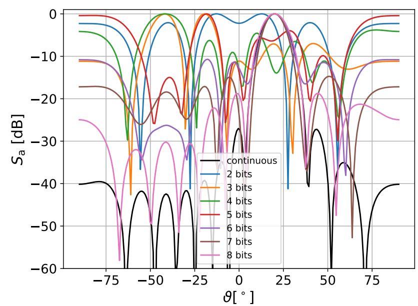
(a) Rectangular window



(b) Hamming window

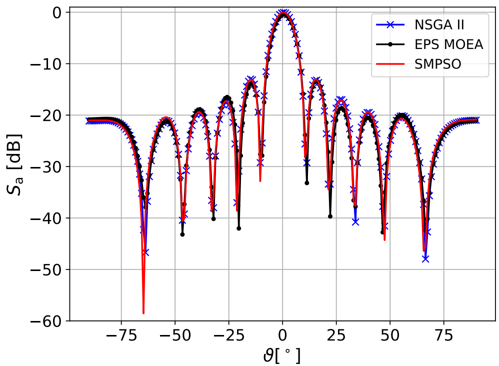


(c) Dolph-Chebychev -30 dB

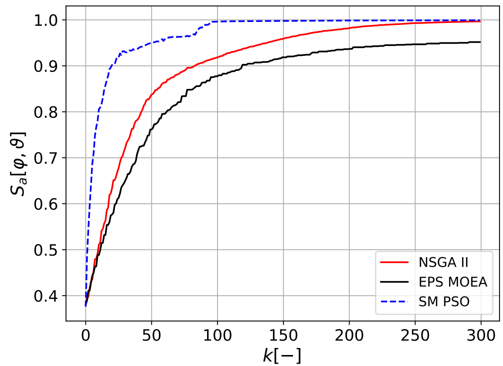


(d) Schelkunoff's zero-placement

Multi-criterial optimization



(a) Array factor



(b) Convergence

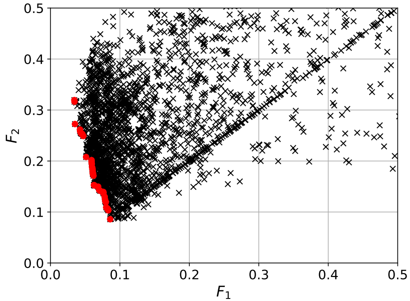
Obrázek. Comparison of algorithms NSGA II, EPS MOEA, SMPSO

Multi-criterial optimization

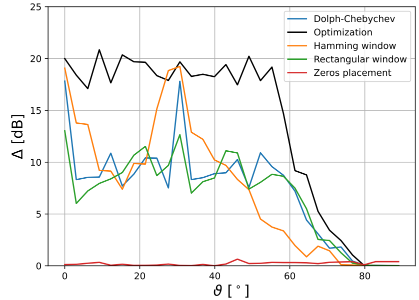
- First objective function - shape of radiation pattern (array factor)
- Second objective function - influence of quantization, measure of robustness

$$\mathcal{F}_2 = \|\text{grad}\,(\mathcal{F}_1)\|$$
$$\mathcal{F}_2 = |\mathcal{F}_1 - \bar{\mathcal{F}}_1|$$

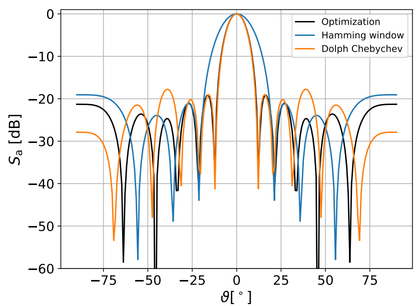
- Other objective functions? Efficiency, beam width, . . .



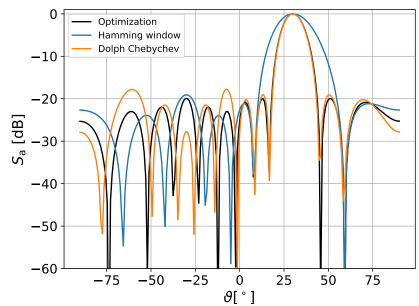
(a) Pareto front



(b) 3-bit quantization



(c) Comparison of array factors obtained by different methods with 3-bit quantized excitation for $\vartheta = 0^\circ$



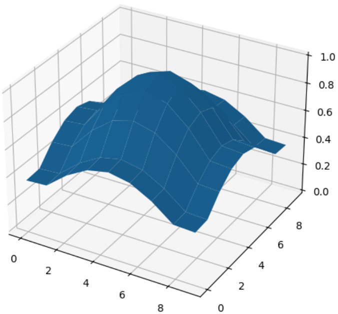
(d) Comparison of array factors obtained by different methods with 3-bit quantized excitation for $\vartheta = 30^\circ$

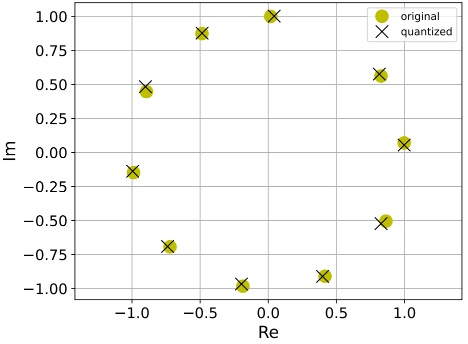
Results

- We can obtain result which is more resistant against quantization errors than standard synthesis methods.
- The number of evaluations of the goal function is to high (200 individuals in one generation at least 100 generations).
- What we can do?

OPTIMIZATION OF OPTIMIZATION

- Magnitudes and phases on particular patches are not arbitrary. There are symmetries

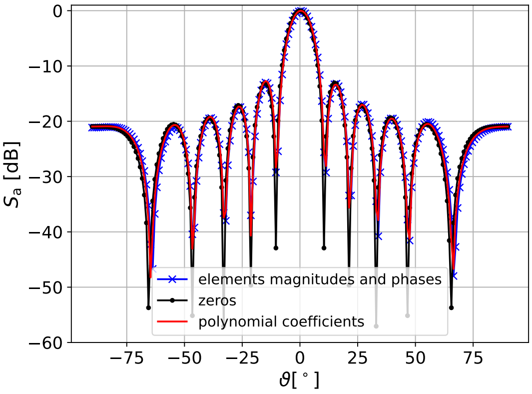




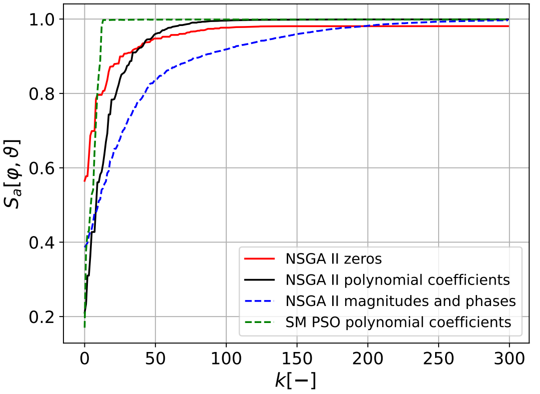
- The matrix of excitation can be obtained as tensor product of two polynomial We can optimize coefficients of these two polynomials or roots of these polynomials.

$$\boldsymbol{I} = \boldsymbol{a} \otimes \boldsymbol{b} \tag{13}$$

Optimization of coefficients and zeros



(a) Array factor



(b) Convergence

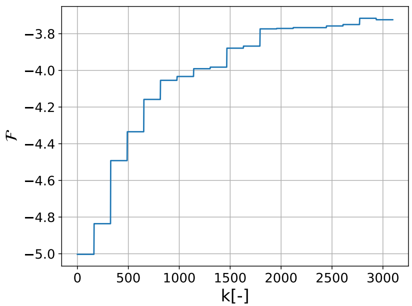
TRUSTED REGION BAYESIAN OPTIMIZATION - TURBO

- Trust region strategy to handle high dimensional problem.
- Hyperrectangle $\mathcal{X}$ centered at the best solution found so far.
- The objective function f is modeled using Gaussian Process (GP)

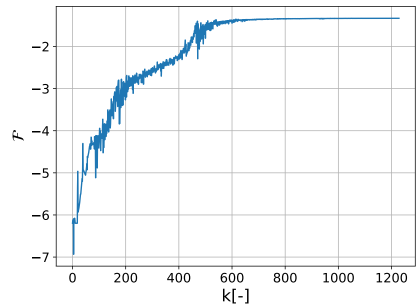
$$f(x) \approx \mathcal{GP}(\mu, K(x, x'))$$

- Acquisition Function - Expected Improvement (EI)

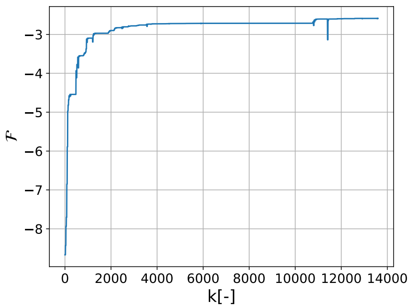
Competition TurBo vs Others



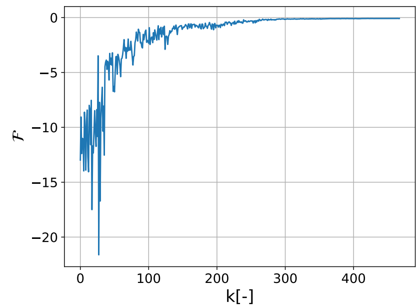
(a) BFGS



(b) Nelder-Mead



(c) Conjungent Gradients



(d) TurBo

Conclusions

- Calibration remain challenging problem.
- Trust Regions Bayesian optimization seems to be best choice in case of single objective optimization.
- There are project for multi-objective optimization within BoTorch.
- Robust design.
- Sparse arrays, nonuniform arrays.

Thank You for Your Attention